Advanced Mathematical Perspectives 1

Lecture 14: Higher-dimensional Random Walks

Matthew Roughan

<matthew.roughan@adelaide.edu.au>

www.maths.adelaide.edu.au/matthew.roughan/notes/AMP1/

School of Mathematical Sciences,
University of Adelaide

THE UNIVERSITY
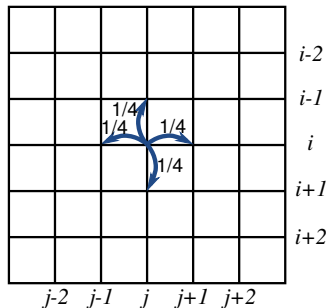*of* ADELAIDE

ACEMS
AUSTRALIAN RESEARCH COUNCIL CENTRE OF EXCELLENCE FOR
MATHEMATICAL AND STATISTICAL FRONTIERS

Section 1

2D Random Walks

A Simple 2D Random Walk

- Imagine moving around a square grid
  - from (i, j)
  - at each time step, move up, down, left or right
  - each possible jump has probability $1/4$

Random Walk Mathematics

Take a series of *vector random variables* $\{X_k\}$ for $k = 1, 2, \ldots$ defined by
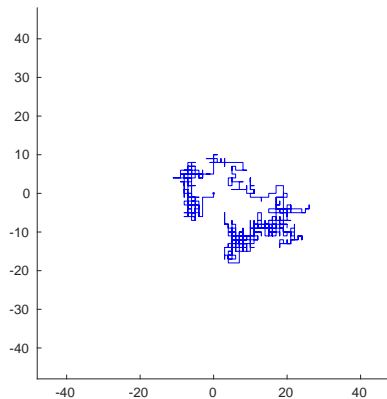
$$X_k = \begin{cases} (1, 0), & \text{with probability } 1/4, \\ (-1, 0), & \text{with probability } 1/4, \\ (0, 1), & \text{with probability } 1/4, \\ (0, -1), & \text{with probability } 1/4. \end{cases}$$

Now we could describe the state of our random walk at time n as a random variable $S_n = (i, j)$, defined by $S_0 = (0, 0)$ and
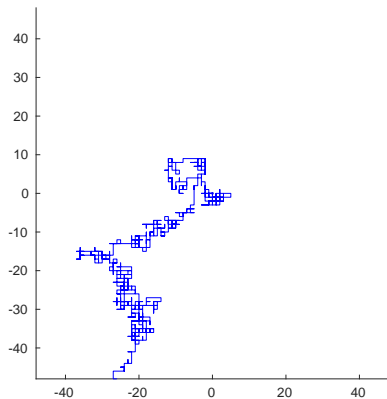
$$S_n = \sum_{k=1}^{n} X_k$$

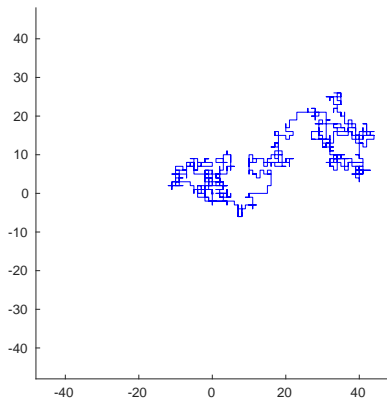This is a very common type of random process, and often analysed.
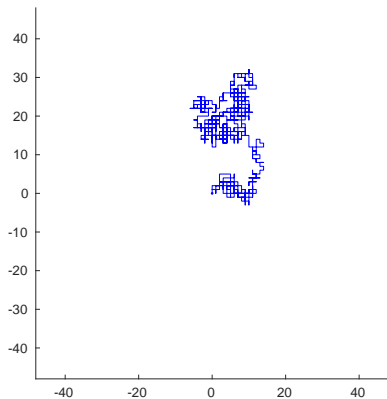
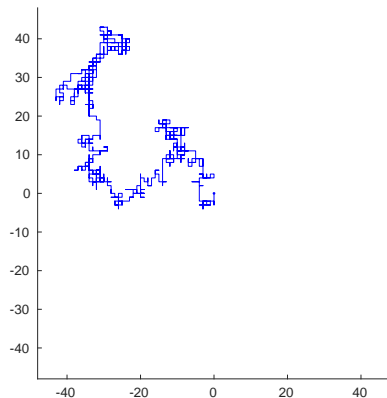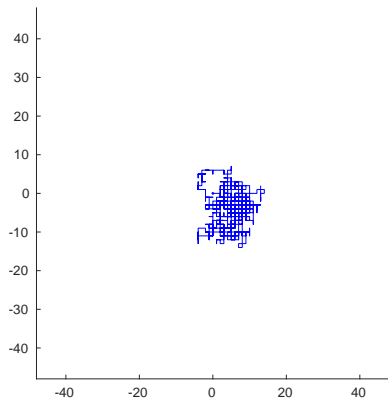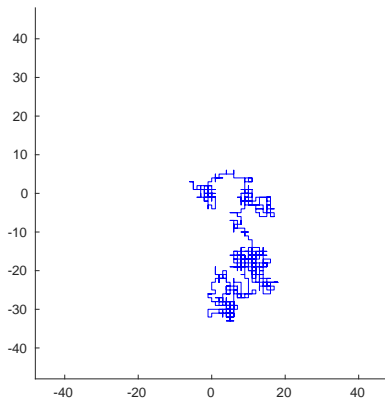# Example Random Walks in 2D

Example Random Walks in 2D

Example Random Walks in 2D

Example Random Walks in 2D

Example Random Walks in 2D

Example Random Walks in 2D
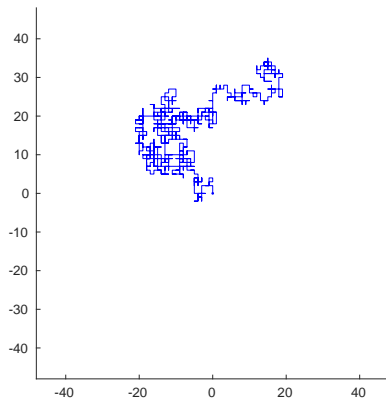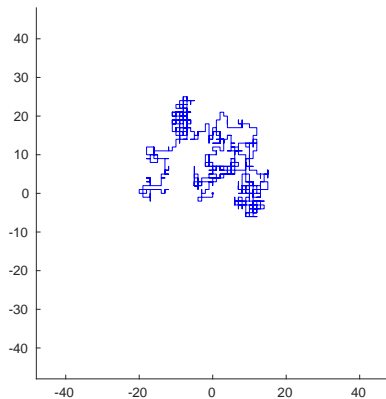
Example Random Walks in 2D

Example Random Walks in 2D

Example Random Walks in 2D

Example Random Walks in 3D

Example Random Walks in 3D

Example Random Walks in 3D

Example Random Walks in 3D
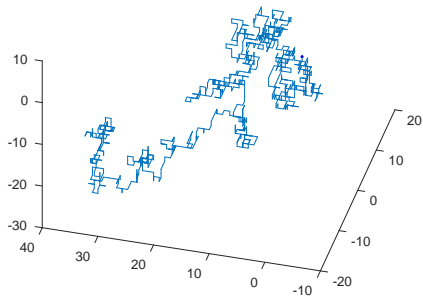
Example Random Walks in 3D
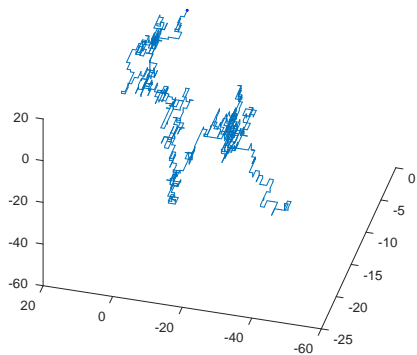
Example Random Walks in 3D
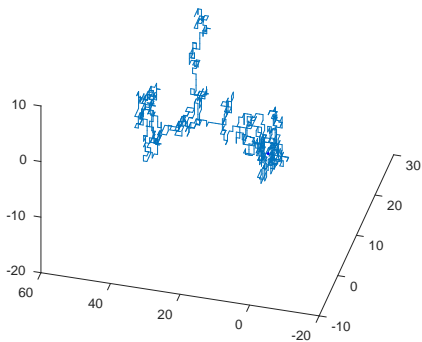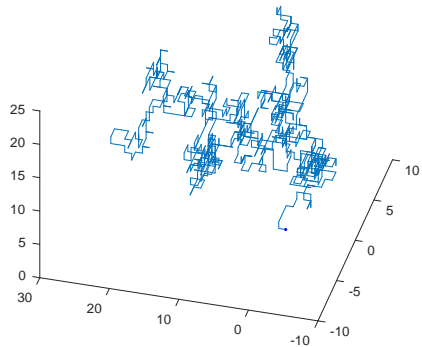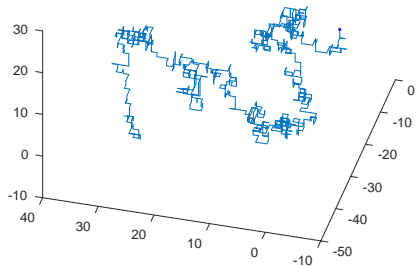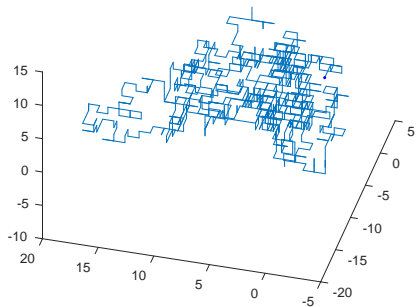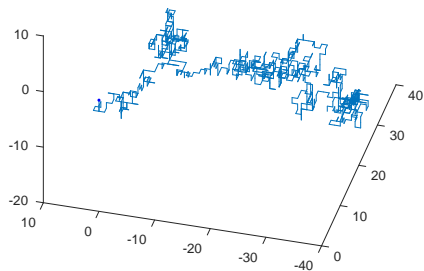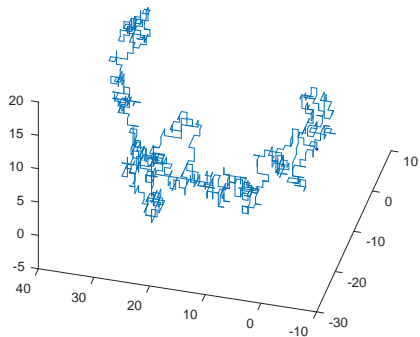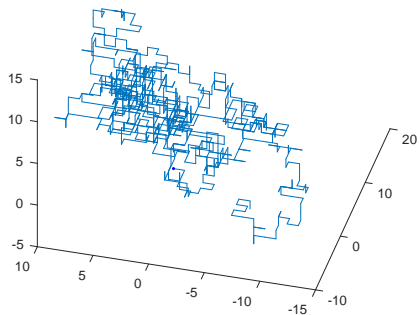
Example Random Walks in 3D

Example Random Walks in 3D

Example Random Walks in 3D

Application: Google's PageRank – a random walk on a network

- We can perform a random walk on any network of points
- Imagine the WWW as a large network, we would like to find the most important pieces (when searching)
- Imagine a person following random links, the amount of time he spends (on average) at a particular page might give some evidence for how central it is to the network.
- This idea is a key part of Google's PageRank algorithm, though they implement it in a more clever way

Random Walks in Art

Many natural phenomena are modelled as variants of random walks, but they also get used in art:

- Antony Gormley's Quantum Cloud is a large sculpture based on a 3D random walk.
- Simon Ingram's Random Walk Machine uses random walks underlying the generation (by machine) of paintings.
- Marius Lehene, Random Walk With Drift
- Chromata

Further reading I

Paul C. Bressloff, *Stochastic processes in cell biology*, ch. Diffusion in Cells: Random Walks and Brownian Motion, Springer, 2014, http://www.springer.com/gp/book/9783319084879.

Sheldon Ross, *Introduction to probability models*, Academic Press, 2010.