Communications Network Design

lecture 05

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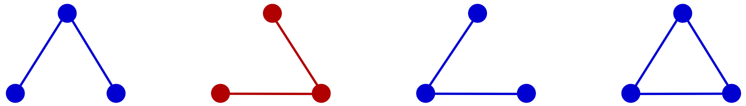
Optimization: 1000 foot view

Its helpful for us to talk a little about optimization techniques before we start. We also presenta little notation.

Note that this course assumes that you already know something about optimization. If you don't, you'd better come and talk to me about it so I can assing you some extra reading to catch up.

Simple example

Three node network has three acceptible designs:



Cost	2.2	2.0	2.4	3.1

- 4 possible network designs
  - associated costs have been worked out for each
- easy to choose the second network as the cheapest

Bigger problems

- Node with N nodes
  - for small N we can evaluate all designs, and choose the best
- But $2^{N(N-1)/2}$ possible network designs
  - some aren't practical
  - but we still have to check that
- Even for $N = 20$ we can't evaluate all of these
  - at least not in the life-time of the Universe

Optimization

Optimization is about building automated methods for finding optima of such problems

- needs to work quickly (enough)
  - planning horizon
  - management requirements
  - size of the problem
- ideally attains provably best solution
  - can't always do this (in reasonable time)
  - our problems are often NP-hard
  - need heuristic (rule of thumb) methods
  - often this isn't a big issue:
    - ⋆ look at all the approximation we already made

High-level view of problems we have

problem	planning horizon	typical goal	variables	common constraints
network design	months	min. cost	capacities	traffic
traffic engineering	days	min. congestion	weights	network design
routing	seconds	min. delay	routes	weights + network

we'll start from back to front (with routing)

Meta-heuristics

High-level view of heuristics

- Greedy
- Gradient Descent
- Branch and Bound
- Simulated Annealling
- Genetic Algorithms

We will often need to use specific properties of a problem in order to make the above practical.

Simple Set Notation

membership	$\omega \in U$	$=$	ω is in U
subset	$L \subset U$	$=$	if $\omega \in L$, then $\omega \in U$
intersection	$L \cap U$	$=$	$\{\omega \mid \omega \in L$ and $\omega \in U\}$
union	$L \cup U$	$=$	$\{\omega \mid \omega \in L$ or $\omega \in U\}$
set difference	$L \backslash U$	$=$	$\{\omega \mid \omega \in L$ and $\omega \notin U\}$
empty set	ϕ	$=$	$\{\}$
for all	$\forall \omega$		do something for all ω
count	$\lvert U \rvert$	$=$	the number of elements of U

The above is an incomplete listing: there are many other possible heuristics (e.g. tabu search), but these are the main ones used here.

Where possible we will take advantage of the optimization you already know: for instance linear programming.

Heuristic doesn't mean "bad". There are some, (e.g. Dijkstra is a greedy algorithm) that are actually guaranteed to find the true optimal solution fairly quickly.

Optimization Notation

We can write an optimization problem different ways

- (1) write it out in full

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{array}$$

- (2) shorter form

$$\max\{\mathbf{c}^T\mathbf{x} \mid A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0\}$$

- (3) even shorter form

$$\underset{A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0}{\text{argmax}} \; \mathbf{c}^T\mathbf{x}$$

Other Notation

I usually use

- lower case for scalars, e.g., x
- lower-case boldface for (column) vectors, e.g., $\mathbf{x}$
- upper-case for matrices, e.g., A

When I write $\mathbf{x} < \mathbf{b}$ I mean every element of $\mathbf{x}$ is less than its corresponding element in $\mathbf{b}$, so

$$x_i < b_i, \;\; \forall i$$

and similarly for fo relational operators, e.g., $\leq$, $\geq$, ...

$$\begin{aligned} \underset{x \in L}{\text{argmax}} \, f(x) &= \text{the value of } x \in L \text{ that maximizes } f(x) \\ &= \hat{x} \text{ such that } f(\hat{x}) \geq f(x), \; \forall x \in L \end{aligned}$$

$$\begin{aligned} \underset{x \in L}{\text{argmin}} \, f(x) &= \text{the value of } x \in L \text{ that minimizes } f(x) \\ &= \hat{x} \text{ such that } f(\hat{x}) \leq f(x), \; \forall x \in L \end{aligned}$$

References