Assignment 2: Due Monday August 26th at 3pm.

Assignments to be handed in through MyUni. Please ensure written assignments are clearly legible. Typed assignments are preferred. Some help may be given in practicals to help get you started with Overleaf/LaTeX in order to present your work well.

This weeks assignment is to write a MATLAB program to perform the Simplex Algorithm (Phase II).

Do not be tempted to skip this, as later assignments will depend on your Simplex code.

To help, I will provide some structure to the task. In particular, using `functions` to *modularise* your code can make it easier to design and test. You can (and should) test each component separately. Practical 2 will give you some help defining MATLAB functions. MATLAB Grader is set up to help you with testing.

Assume the problem is given in the form:

$$\begin{aligned} \max \quad & z = \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where $\mathbf{b} \geq 0$, so that we can convert directly to standard equality form using slack variables, and the resulting Tableau will be in feasible canonical form, so we can skip Simplex Phase I.

1. Start by writing a function called `Mout = pivot(Min,i,j)`, as in your practical. It should take 3 input arguments: an array M_{in}, representing a tableau and two natural numbers i and j. It should check them, and return the array M_{out} after pivoting M_{in} at the point (i, j), if this is a valid pivot.
2. Write a function `at_end` that takes a tableau M as input, and returns true or false, depending on whether Simplex has reached its final step. Be careful about floating point errors!
3. Write a function `choose`, which takes a tableau M as input, and returns natural numbers i and j, which give the *entering* pivot row and column (the next pivot location).
   When you can't find a viable leaving variable, your code needs to indicate this by returning i and j values $i = j = \infty$. (**Hint:** floating point numbers can have the value `Inf`).
   This function could be integrated with the previous function if you do it cleanly. But Grader tests will be set up to test each independently.
4. Write the function `M = construct(A, b, c, z0)` to construct the Phase II Tableau. Input vectors should be column vectors and sizes should match. Once again, check your inputs, for instance sizes of vectors, and that $\mathbf{b} \geq \underline{0}$.
5. Use the above to implement a function called `simplex` that performs Simplex Phase II, and returns the value of $\mathbf{x}$ that maximises z. See below for inputs and outputs.

All of the above are, in principle, quite easy programming tasks. However, you must remember to be careful about several features:

- unless otherwise stated, all vectors are to be input and returned as column vectors;
- floating point arithmetic is not exact: *e.g.,* do not expect tests such as $x == y$ to work;
- you should make sure you use robust rules to choose pivot locations; and
- efficiency and robustness: I may use a large test problem, or a problem with unexpected values (**Hint:** Grader problems are not a complete test set) and your code must be able to handle these cases. Create some test problems of your own!

You can test your code in MATLAB itself, or in MATLAB Grader. Your assignment will be assessed by

- me reading the code for style; and
- by automatically running your functions against a set of test problems: marks will be lost for incorrect answers.

The latter requires that you hand it up in a particular form. Assignments that fail to adopt this form will receive zero marks. The assigment *must* be in a `.zip` file, with the name

studentnumber`_simplex.zip`

The ZIP file *must* contain a folder called `assignment2` (please ensure that case is correct, and that this is a folder, not a vomit of files), containing the functions listed above, *i.e.,* it must have files

```
assignment2/pivot.m
assignment2/at_end.m
assignment2/choose.m
assignment2/construct.m
assignment2/simplex.m
```

The top-level function should be called as follows:

```
[output_flag, x, z] = simplex(A, b, c, z0);
```

where the inputs are the elements of the problem described above, and the output is a flag to indicate how the code finished, and the optimal $\mathbf{x}$ and z values. All vectors are to be entered and output as column vectors.

Extra hints:

- If your code does not terminate or crashes, **you will receive zero**.
- Your program should check that the inputs are valid, *i.e.,* that the size of the matrices and vectors are consistent, and return a meaningful error if they are not. You may want to use the `assert` function. This will be tested.
- There are three possible output cases, indicated by the value of `output_flag`
  - you find a feasible, optimal point: `output_flag = 0`
  - the problem is infeasible: `output_flag = 1`
  - the problem is unbounded: `output_flag = 2`
- Your code should have NO side effects. It should not print out any values or make any plots. Please be careful about this. The scripts are automatically marked, and any extra outputs may cause the marking program to crash, and **you will receive zero.**
- Please ensure that your code follows standard good practice regarding comments, variables names, stucture and so on: marks will be subtracted for bad programming style.
- Plagiarism (copying your code from other students, or from the Internet) will be taken *very* seriously. Please ensure that you only hand in your own work – it will be checked.

These assignment will be handed in electronically through MyUni. MATLAB grader is only used to give you help.