Tutorial 3

Make sure you prepare these BEFORE the class.

Solutions will be handed out at the tutorial. They will not be put on MyUni.

1. **Translation:** The following is called the *Transportation problem.*

   There are three warehouses at different cities: Sydney, Melbourne and Adelaide. They have 250, 130 and 235 tonnes of paper accordingly. There are four publishers, in Sydney, Melbourne, Brisbane and Hobart. They ordered 75, 230, 240 and 70 tonnes of paper to publish new books. There are the following costs (in tens of dollars) of transportation of one tonne of paper:

From / To	Sydney	Brisbane	Melbourne	Hobart
Sydney	15	20	16	21
Melbourne	25	13	5	11
Adelaide	15	15	7	17

   The idea is to find a *transportation plan* such that all orders will be met and the transportation costs will be minimized.

   (a) Formulate the problem as a Linear Program.

   (b) How could you change this if the total paper ordered was less than the total supply?

   (c) What if only whole tonnes of paper can be delivered?

   Hints: remember to look for three things:

   1. the variables: here we could use a 2D array of variables x_{ij} (remember to define what these mean);
   2. the objective (the thing you want to maximise or minimise); and
   3. the constraints (here there will be a constraint for each warehouse and each publisher).

2. **Calculations:** Graph the constraints of the following problem and its corresponding dual.

$$
\begin{array}{llrrl}
\text{(P)} & \max \quad z = & x_1 & -x_2 & \\
& \text{s.t.} & 2x_1 & +x_2 & \leq 2 \\
& & -x_1 & -x_2 & \leq 1 \\
& & x_1, & x_2 \geq 0 &
\end{array}
$$

   Intepret the sketches and their relationship.

3. **Proof of the week:** There are more general duality conditions than presented in Lecture 9. One example is presented below.

   Assuming that we had the primal problem

$$
\begin{array}{rrcl}
\max & z & = & \mathbf{c}^T\mathbf{x} + z_0 \\
\text{such that} & A\mathbf{x} & \leq & \mathbf{b} \\
\text{and} & \mathbf{x} & \geq & 0
\end{array}
$$

   its dual is

$$
\begin{array}{rrcl}
\min & w & = & \mathbf{b}^T\mathbf{y} + z_0 \\
\text{such that} & A^T\mathbf{y} & \geq & \mathbf{c} \\
\text{and} & \mathbf{y} & \geq & 0
\end{array}
$$

   Prove that $w \geq z$ for any feasible points $\mathbf{x}$ and $\mathbf{y}$ of the two respective problems.

4. **Complexity calculation:** Consider polynomial evaluation: that is, we want to evaluate the degree-n polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where $a_n \neq 0$.

(a) Compute the complexity of an algorithm that calculates the above by brute force, *i.e.,* by performing the calculations as written, and computing powers using multiplication.

(b) Now find an $O(n)$ algorithm.