Transform Methods & Signal Processing
Class Exercise 1:
Do Not Hand up

Matthew Roughan
<matthew.roughan@adelaide.edu.au>

The goal of this class exercise is to assess your grasp of the assumed knowledge for this course (Level II Fourier Series and Differential Equations or the equivalent). No marks will be assigned, but you need to make sure that you can do all of these exercises. If not, please come and see me.

1. **Trigonometry:**
   (a) simplify the following
      i. $\cos^2(x) + \sin^2(x)$
      ii. $\sin(x + \pi/2)$
      iii. $\sin(\alpha x + \pi)$
      iv. $\cos(x - 2\pi)$

   (b) write the following without any products of cosines or sines
      i. $2\cos(nx) \times \cos(mx)$
      ii. $2\sin^2(\theta)$

2. **Differentiation and Integration:**
   (a) Differentiate the following functions
      i. $f(x) = e^{\alpha x}$
      ii. $f(x) = x^2 \cos(-x)$
      iii. $f(x) = \sin(\ln(x))$
      iv. $f(x) = \sum_{i=0}^{\infty} x^i$, for $|x| < 1$
      v. $x^{1/\ln(x)}$

   (b) Find the integrals
      i. $\int e^{\alpha x}\, dx$
      ii. $\int_0^1 x e^{-ix}\, dx$
      iii. $\int_{-\infty}^{\infty} e^{-\pi t^2} e^{-i2\pi st}\, dt$
      iv. $\int_{-\pi}^{\pi} \cos(nx)\, \cos(mx)\, dx$
      v. $\int_0^2 \int_0^4 (x^2 - y^2)\, dx\, dy$

3. Co-ordinate systems: Take circular co-ordinates r, θ such that

$$x = r\cos\theta, \quad y = r\sin\theta$$

(a) draw a picture to illustrate this coordinate system

(b) calculate the Jacobian J of the coordinate transform

4. Complex numbers:

(a) simplify $(2+i) \times (1-3i)$

(b) find the imaginary part $\Im\left(\frac{2+i}{5-3i}\right)$

(c) find the complex conjugate of $a + bi$

(d) draw a picture of the set $|z| \leq 1$

(e) find all of the values of $\sqrt[4]{-1}$

(f) for complex number z find $\frac{d}{dz}e^z$

(g) show $e^{z_1}e^{z_2} = e^{z_1+z_2}$ where $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$

5. Fourier series:

(a) which of the following functions *odd*, *even* or *neither*

$$x^2, \quad e^x, \quad e^{|x|}, \quad x\sin(nx)$$

(b) given a periodic function (period 4), defined by $f(x) = x$ on the interval $[-2, 2]$

i. draw this function over the interval $[-6, 6]$

ii. obtain the Fourier series representation of $f(x)$.