Transform Methods & Signal Processing
Class Exercise 2:
Hand in before before lecture, 24th August

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Note, questions marked by a (*) are harder than normal questions, and are for masters students. Bonus marks may be awarded to other students who solve these.

1. [2 marks] Derive (from the definition of FT) the continuous Fourier transform of the following functions

(a) $f(t) = r(t)$, where $r(t)$, where r is a rectangular pulse of unit width.

2. [8 marks] Derive the continuous Fourier transform of the following functions (using any results given in lectures)

(a) $f(t) = Ae^{-\pi(at)^2}e^{-i2\pi s_0 t}$

(b) $f(t) = \cos(2\pi s_0 t) * r(t)$, where $r(t)$ is a rectangular pulse of unit width.

(c) $f(t) = \frac{d^2}{dt^2}\text{sinc}(t)$

3*. [5 marks] Prove that the continuous Fourier Transform, and Inverse Fourier transform, are really inverse operators for all functions in $L^1(I\!R)$, e.g. show that

$$\mathcal{F}^{-1}\{\mathcal{F}\{f(t)\}\} = f(t)$$

for all functions for which $\int_{-\infty}^{\infty} |f(t)| \, dt < \infty$.

[Hint: Note to use Fubini's theorem, one needs finiteness of the integrals. To ensure this, multiply the signal by a Gaussian, and then relax the Gaussian by increasing its standard deviation, taking the limits. Be careful in taking limits of integrals (e.g. you need the dominated convergence theorem).]