Communications Network Design

lecture 13

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Branch and bound (cont)

The simple branch and bound solution shown previously is rather naive. It doesn't take advantage of the structure of the problem. We show how branch and bound can be applied to the budget constraint model, by showing the relationship with the **knapsack problem**. The useful result we get is the **Dionne-Florian** lower bound, which can be used in bounding.

Branch and Bound

- Branch and bound subsumes many specific approaches, and allows for a variety of implementations.
- partition, sampling, and subsequent lower and upper bounding procedures: these operations are applied iteratively to the collection of active ('candidate') subsets within the feasible set D
- Branch and bound methods typically rely on some a priori structural knowledge about the problem.

Budget Constraint Model

$$
\begin{array}{llrll}
(P') & \min & C(\mathbf{f}) & = \sum_{e \in L} \alpha_e f_e & \\
& \text{s.t.} & f_e & = \sum_{\mu : e \in \mu} x_\mu & \forall e \in E \\
& & \sum_{\mu : \mu \in P_k} x_\mu & = t_k & \forall k \in K \\
& & \sum_{e \in E} \beta_e z_e & \leq B & \\
& & x_\mu & \geq 0 & \forall \mu \in P \\
& & z_e & = 0, \text{ or } 1 & \forall e \in E
\end{array}
$$

$$
z_e = \begin{cases} 1 & \text{if link } e \in L \text{ (i.e. we use } e) \\ 0 & \text{if link } e \notin L \text{ (i.e. we don't use } e) \end{cases}
$$

Budget Constraint Model

The cost is now $C(\mathbf{f}) = \sum_{e \in L} \alpha_e f_e = \sum_{k \in K} t_k \hat{l}_k(L) = v(L)$

- The network design is determined by the choice of L, the links we will use, which in turn determines the routes and then the link loads, so that the cost is really a function of L, which we write $v(L)$ here.
- The cheapest possible network will have all links present, i.e., $v(E)$ is the lowest cost
  - link creation costs have been shifted into (budget) constraint
  - any missing links might cause rerouting, which could in turn increase the cost

Budget Constraint Model Bounds

- $v(E)$ is the lowest cost
- what happens if we remove link $e = (i, j)$
  - the traffic $t_{i,j}$ must be rerouted on a non-direct route
  - hence, higher cost (or at least no lower)
  - take $d_{(i,j)}$ to be the cost of rerouting traffic $t_{i,j}$ because link $e = (i, j)$ is removed from the link set

$$d_{(i,j)} = \left[\hat{l}_{(i,j)}\Big(E \setminus (i,j)\Big) - \alpha_{(i,j)}\right] t_{i,j}$$

- So for any link set $L \subseteq E$,

$$v(L) \geq v(E) + \sum_{e \notin L} d_e$$

Budget Constraint Model Bounds

$$v(L) \;\geq\; v(E) + \sum_{e \notin L} d_e$$

for feasible networks, i.e. $\sum_{e \in L} \beta_e \leq B$

- thus we can get a lower bound on the cost of all feasible networks
- Note, in B&B on simple LPs, we were finding upper bounds for maximization from relaxations
  - here we are finding minimums (costs)
  - hence we get **lower** bounds from our relaxations
  - so the above is doing the right thing for a relaxation

Budget Constraint Model Bounds

$$\begin{aligned} v(L) &\geq v(E) + \sum_{e \notin L} d_e \\ &= v(E) + \sum_{e \in E} (1 - z_e) d_e \\ &\geq v(E) + \sum_{e \in E} d_e - w \end{aligned}$$

for all feasible solutions L such that $\sum_{e \in L} \beta_e \leq B$ and where $w = \sum_{e \in E} d_e z_e$

- the lower bound on $v(L)$ will be smallest when w is largest.
- we need to look for the maximum value of w, e.g. $\max\{\sum_{e \in E} d_e z_e | \sum_e \beta_e z_e \leq B, z_e = 0 \text{ or } 1\}$

Budget Constraint Model Bounds

- so we have a new IP to solve
  $\max\{\sum_{e\in E} d_e z_e \,|\, \sum_e \beta_e z_e \leq B, z_e = 0 \text{ or } 1\}$
- this is a **knapsack problem** [1]
- we can do the standard relaxation to a LP, to get the problem

$$\text{LP} \begin{cases} \text{maximize} & w^R = \sum_{e\in E} d_e z_e \\ \text{subject to} & \sum_e \beta_e z_e \leq B \\ & 0 \leq z_e \leq 1 \end{cases}$$

- remember that it is a relaxation of the IP, so

$$w^R \geq w$$

  so it is an upper bound on w, and so gives us a lower bound on $v(L)$

Knapsack problem

Integral knapsack problem

- we have a knapsack (backpack) with finite volume B
- we want to fit as much useful stuff into it as possible
  - maximize the value of the items contained
- each item e
  - has a volume β_e
  - has a value d_e
- if we include the item, we say $z_e = 1$
  - otherwise $z_e = 0$
- maximum value is obtained when we find $\max\{\sum_{e\in E} d_e z_e | \sum_e \beta_e z_e \leq B, z_e = 0$ or $1\}$

Knapsack problem

Fractional knapsack problem

- as noted earlier, the integral knapsack problem is NP-hard
- so we relax the problem to a linear program $\max\{\sum_{e \in E} d_e z_e | \sum_e \beta_e z_e \leq B, 0 \leq z_e \leq 1\}$
- call this the fractional knapsack problem
  - because we are allowed to break items up into fractions (given by z_e)
- this problem is easier to solve than even many other LPs [2]

Fractional knapsack solution

- rank all links $e \in E$ in order such that

$$\frac{d_{e_1}}{\beta_{e_1}} \geq \frac{d_{e_2}}{\beta_{e_2}} \geq \cdots \frac{d_{e_{|E|}}}{\beta_{e_{|E|}}}$$

- $\frac{d_e}{\beta_e}$ can be thought of as the **unit worth** of e
  - remember analogy of d_e as value, and β_e as volume
- find the largest integer k such that

$$\sum_{i=1}^{k} \beta_{e_i} \leq B$$

- fill the knapsack with items of most unit worth first.
  - until we reach k
  - then we use a fraction of the next item

Fractional knapsack solution

- the solution is

$$z_{e_i} = \begin{cases} 1 & \text{for } i = 1, 2 \ldots, k \\ \dfrac{B - \sum_{i=1}^{k} \beta_{e_i}}{\beta_{e_{k+1}}} & \text{for } i = k+1 \\ 0 & \text{for } i \geq k+2 \end{cases}$$

- complexity of the solution is
  - $O(|E| \log |E|)$ for the sorting operation
- it can be done faster by a weighted median search [1, p.398] which takes time $O(|E|)$

Dionne-Florian lower bound

Now, the lower bound on the cost for a feasible design L will be

$$
\begin{aligned}
v(L) &\geq v(E) + \sum_{e \notin L} d_e \\
&= v(E) + \sum_{e \in E} (1 - z_e) d_e \\
&= v(E) + \sum_{i=k+2}^{|E|} d_{e_i} + d_{e_{k+1}} \left\{ 1 - \left[\frac{B - \sum_{i=1}^{k} \beta_{e_i} z_{e_i}}{\beta_{e_{k+1}}} \right] \right\} \\
&= v(E) + \sum_{i=k+1}^{|E|} d_{e_i} - \frac{d_{e_{k+1}}}{\beta_{e_{k+1}}} \left\{ B - \sum_{i=1}^{k} \beta_{e_i} z_{e_i} \right\}
\end{aligned}
$$

This is called the **Dionne-Florian lower bound**

Branch and Bound: setup

- Let $\overline{\mathbf{z}}^{(m)}$ be a partial solution for $\mathbf{z}$.
  - $\overline{\mathbf{z}}^{(m)}$ is a 0-1 vector of m components, $m \leq |E|$
  - Entries $\overline{z}_e^{(m)}$ in $\overline{\mathbf{z}}^{(m)}$ give the status of links already decided in the design being considered
  - That is

$$\overline{z}_e^{(m)} = \begin{cases} 1 & \Rightarrow \text{ link } e \text{ is } \textbf{included} \text{ the design} \\ 0 & \Rightarrow \text{ link } e \text{ is } \textbf{not included} \text{ the design} \end{cases}$$

Branch and Bound: setup

- Let $D(\overline{\mathbf{z}}^{(m)}) \subseteq E$ be the design corresponding to $\overline{\mathbf{z}}^{(m)}$
  - it already contains the links corresponding to the 1's of $\overline{\mathbf{z}}^{(m)}$
  - it omits links corresponding to the 0's of $\overline{\mathbf{z}}^{(m)}$
  - other links are undecided
- So $D(\overline{\mathbf{z}}^{(m)}) \subseteq E$ and $\overline{z}_e^{(m)} = 0, 1$ for all $e \in D(\overline{\mathbf{z}}^{(m)})$.
- Obviously, if $e \notin D(\overline{\mathbf{z}}^{(m)})$, then the status of link e has not yet been determined, so we need to determine z_e for all $e \notin D(\overline{\mathbf{z}}^{(m)})$.
- This will give a **completion** $\mathbf{z}$ of $\overline{\mathbf{z}}^{(m)}$

Branch and Bound

We can use the D-F lower bound in B&B as follows:

- we are doing a minimization, so we need a lower-bound at each subproblem
- calculate lower bounds on the cost of a design $D(\bar{\mathbf{z}})$ as follows:
  - given $\bar{\mathbf{z}}$, determine a completion of $\bar{\mathbf{z}}$ using the knapsack problem approach above
- Suppose for ease of reference,

$$e_i \notin D(\bar{\mathbf{z}}^{(m)}), \qquad \text{for } i = 1, 2, \ldots, |E| - m$$

and the e_i are listed by **decreasing relative worth**. Then a lower bound on the cost of $D(\bar{\mathbf{z}}^{(m)})$ is $b(\bar{\mathbf{z}}^{(m)})$

Branch and Bound

$$b(\overline{\mathbf{z}}^{(m)}) = \begin{bmatrix} \text{original} \\ \text{cost with} \\ \text{all links} \\ \text{i.e. } v(E) \end{bmatrix} + \sum \begin{bmatrix} \text{changes in cost} \\ \text{for rerouting} \\ \text{loads on links} \\ \text{determined to NOT} \\ \text{be in the design} \\ \text{i.e. } \overline{z}_e^{(m)} = 0 \end{bmatrix}$$

$$+ \left(d_{e_{k+1}} - \frac{d_{e_{k+1}}}{\beta_{e_{k+1}}} \begin{bmatrix} \text{leftover} \\ \text{bit of} \\ \text{budget } B \\ \text{from} \\ \text{knapsack prob.} \end{bmatrix} \right)$$

Branch and Bound

The last part corresponds to rerouting a fraction of load from e_{k+1} because a fraction of the link is "missing" ($z_{e_{k+1}}$ is fractional!). So

$$b(\overline{\mathbf{z}}^{(m)}) = v(E) + \left[\sum_{e:\overline{z}_e^{(m)}=0} d_e + \sum_{i=k+2}^{|E|-m} d_{e_i} \right] + \left[d_{e_{k+1}} - \left\{ B - \sum_{e \in D(\overline{\mathbf{z}})} \beta_e z_e - \sum_{i=1}^{k} \beta_{e_i} \right\} \frac{d_{e_{k+1}}}{\beta_{e_{k+1}}} \right]$$

Then we just apply branch and bound as before, using this bound.

Branch and Bound outline

If $\sum_{e \in E} \beta_e \leq B$, then **STOP**

- the optimal design is the fully meshed network

Otherwise,

Initialise:

- list all links in E in order of decreasing relative weights, $\frac{d_{e_i}}{\beta_{e_i}}$.
- $\mathcal{L} = \mathrm{IP}^0$;
- $D(\overline{\mathbf{z}}) = \emptyset$;
- best-to-date cost $C = \infty$

Branch and Bound outline

At any stage: given a list $\mathcal{L}$ of partial solutions $\{\bar{\mathbf{z}}\}$ and their corresponding lower bounds, $b(\bar{\mathbf{z}})$, select one $\bar{\mathbf{z}} \in \mathcal{L}$ and attempt to fathom it. That is, remove it from $\mathcal{L}$ and

(a) solve the fractional knapsack problem and compute the D-F lower-bound $b(\bar{\mathbf{z}})$

- If this has an integer feasible part solution, it is fathomed. If the cost of the integer solution $C' < C$ then this becomes the best-to-date cost and we update C to C'.
  - we can prune any solutions with $b(\bar{\mathbf{z}}) > C$
- if the solution has lower bound $\bar{\mathbf{z}}$ greater than the best-to-date cost C, then it is fathomed, and we can prune it.
- if infeasible then it is fathomed

Branch and Bound outline

(b) If not fathomed construct two new partial solutions by selecting a link e not determined in $\bar{\mathbf{z}}$ and putting

(i) $\bar{z}_e = 1$

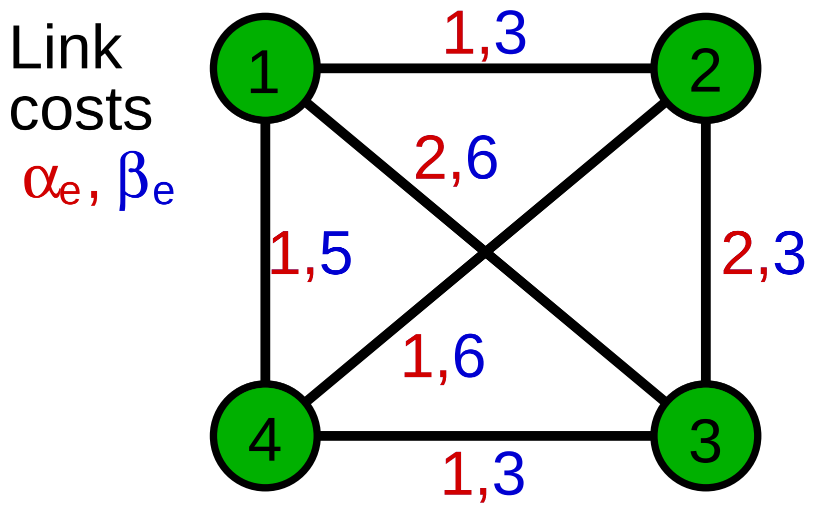
(ii) $\bar{z}_e = 0$

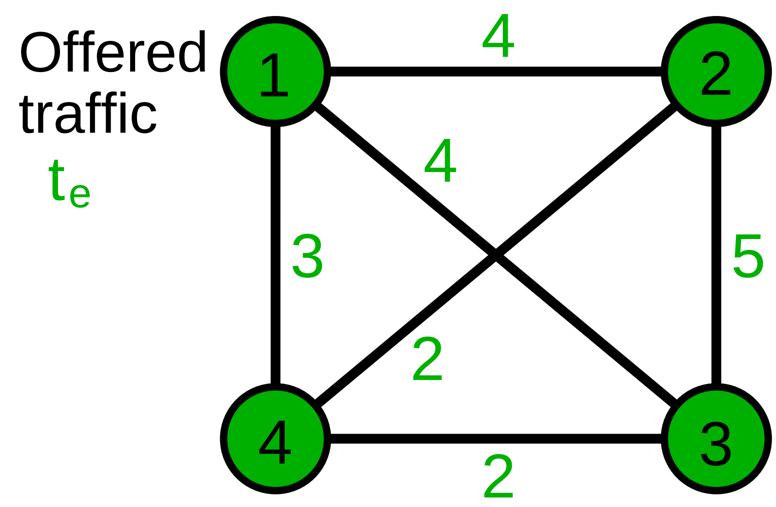
Note: Select e in order of decreasing d_e/β_e

Continue until all partial solutions have been fathomed

Branch and Bound example

The network $G(N,E)$ and data for (α_e, β_e) and offered traffic, t_{pq} (as in Minoux's method example, Lecture 14)





$$C(\mathbf{f}) = \sum_{e \in L} c_e(f_e)$$

$$c_e(f_e) = \alpha_e f_e + \beta_e$$

$$\sum_e \beta_e z_e \leq B = 14$$

Branch and Bound example

- Assume all routing is direct i.e. $f_e = t_k$
- Now $v(E) = \sum_e \alpha_e f_e = 4+3+8+2+10+2 = 29$
- Since $d_e = \left[\hat{l}_\mu(E-e) - \alpha_e\right] f_e$, we have the table:

$e=(i,j)$	α_e	$\hat{\mu}_{ij}(E-e)$	$\hat{l}_{ij}(E-e)$	d_e	d_e/β_e	rank
$(1,2)$	1	$1-4-2$	2	$(2-1).4=4$	$4/3$	1
$(1,3)$	2	$1-4-3$	2	$(2-2).4=0$	0	5
$(1,4)$	1	$1-2-4$	2	$(2-1).3=3$	$3/5$	3
$(2,3)$	2	$2-4-3$	2	$(2-2).5=0$	0	6
$(2,4)$	1	$2-1-4$	2	$(2-1).2=2$	$1/3$	4
$(3,4)$	1	$3-1-4$	3	$(3-1).2=4$	$4/3$	2

Branch and Bound example

Rank all links in order of decreasing $\frac{d_{e_i}}{\beta_{e_i}}$

	edge					
	$(1,2)$	$(3,4)$	$(1,4)$	$(2,4)$	$(1,3)$	$(2,3)$
d_e	4	4	3	2	0	0
β_e	3	3	5	6	6	3
$\frac{d_e}{\beta_e}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{3}{5}$	$\frac{1}{3}$	0	0

Table 1

We will use Table 1 repeatedly in this example.

Branch and Bound example

$\sum_{e \in E} \beta_e = 26 > B = 14$, so can't just use E

Table 1 lists links in order of decreasing $\frac{d_{e_i}}{\beta_{e_i}}$

Initialise:

- $\mathcal{L} = \mathrm{IP}^0$;
- $D(\bar{\mathbf{z}}) = \emptyset$;
- best-to-date cost $C = \infty$

Branch and Bound example

Problem 0: $\bar{\mathbf{z}} = ()$
Knapsack problem:

- count across β_e row until $\sum_{i=1}^{k} \beta_{e_i} \leq B$ and $\sum_{i=1}^{k+1} \beta_{e_i} > B$

  $\beta_{12} + \beta_{34} + \beta_{14} = 11 < 14; \qquad \beta_{12} + \beta_{34} + \beta_{14} + \beta_{24} = 17 > 14.$

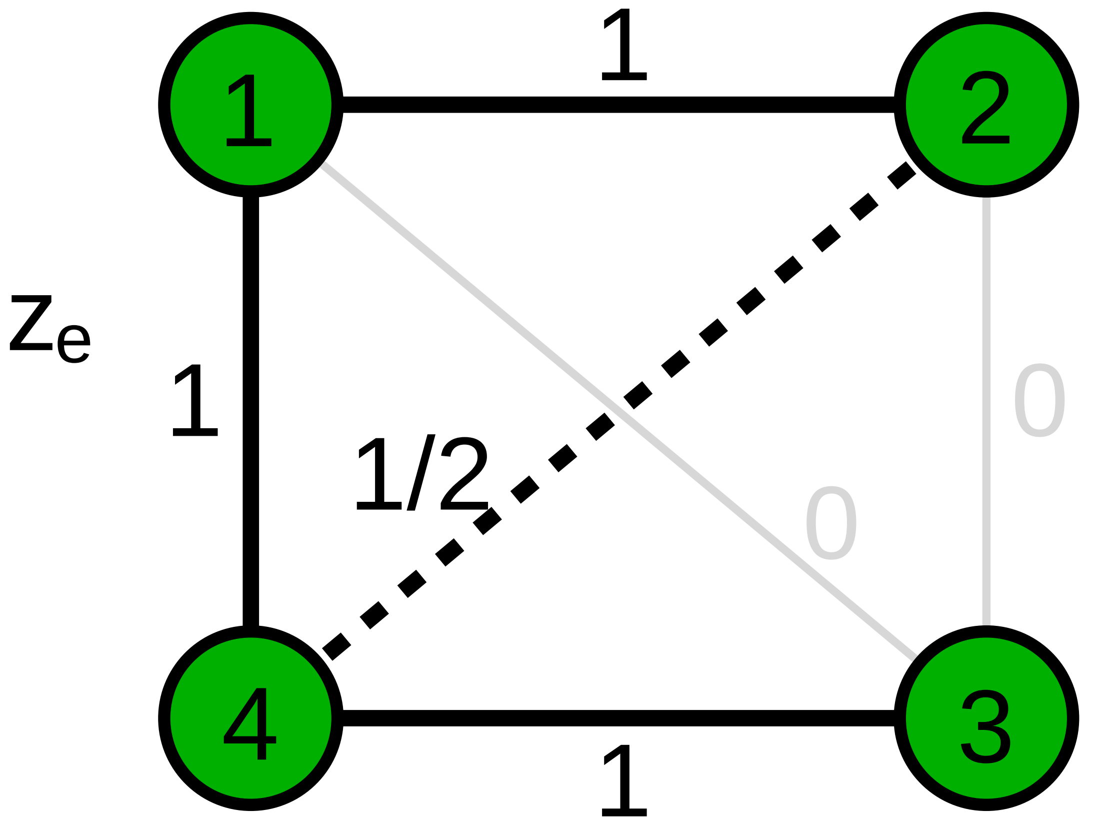
- $k = 3$, and $z = (1, 1, 1, 1/2, 0, 0)$
- solution is not integer feasible

D-F lower bound:

$$
\begin{aligned}
b(\bar{\mathbf{z}}) &= v(E) + [d_{13} + d_{23}] + (1 - \tfrac{1}{2})d_{24} \\
&= 29 + (0 + 0) + \tfrac{1}{2}.2 \\
&= 30
\end{aligned}
$$

Branch and Bound example

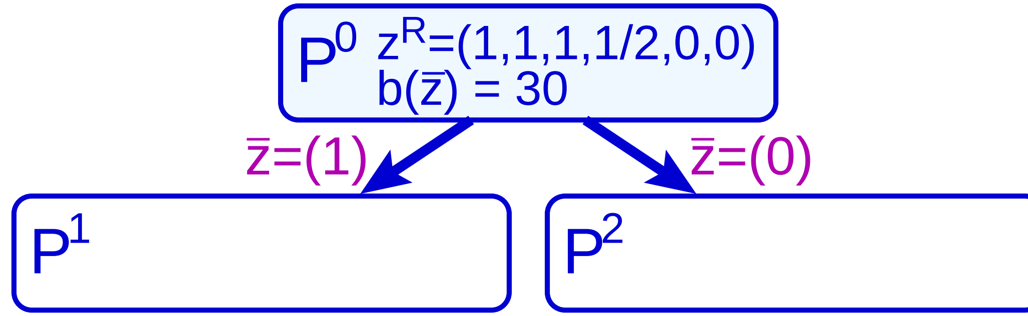
P^0 relaxation solution



Branch and Bound example

- Note that $b(\bar{\mathbf{z}}) > v(E)$ as expected: if you delete links from E, and have to reroute then the operating costs should increase.
- the solution was not integer feasible, so we have to branch into two subproblems
  - P^1: $\bar{\mathbf{z}} = (1)$ (we add the constraint $z_{12} = 1$)
  - P^2: $\bar{\mathbf{z}} = (0)$ (we add the constraint $z_{12} = 0$)
- our list of outstanding subproblems becomes $\mathcal{L} = \{\mathsf{P}^1, \mathsf{P}^2\}$

Branch and Bound example



Branch and Bound example

Problem 1: $\bar{\mathbf{z}} = (1)$

Knapsack problem: exactly the same as problem 0

- solution for P^0 had $z_{12} = 1$, so $\bar{\mathbf{z}} = (1)$ doesn't change the solution at all
- $k = 3$, and $z = (1, 1, 1, 1/2, 0, 0)$
- solution is not integer feasible

D-F lower bound: $b(\bar{\mathbf{z}}) = 30$ (the same as P^0)

- the solution was not integer feasible, so we have to branch into two subproblems
  - P^3: $\bar{\mathbf{z}} = (1, 1)$ (we add the constraint $z_{34} = 1$)
  - P^4: $\bar{\mathbf{z}} = (1, 0)$ (we add the constraint $z_{34} = 0$)

Branch and Bound example

Problem 2: $\bar{\mathbf{z}} = (0)$

Knapsack problem:

- β_{12} is excluded, so only consider columns 2-6
- count across β_e row until $\sum_{i=2}^{k} \beta_{e_i} \leq B$ and $\sum_{i=2}^{k+1} \beta_{e_i} > B$

$$\beta_{34} + \beta_{14} + \beta_{24} = 14$$

- $k = 4$, and $z = (0,1,1,1,0,0)$
- solution is integer feasible, so it is **fathomed**

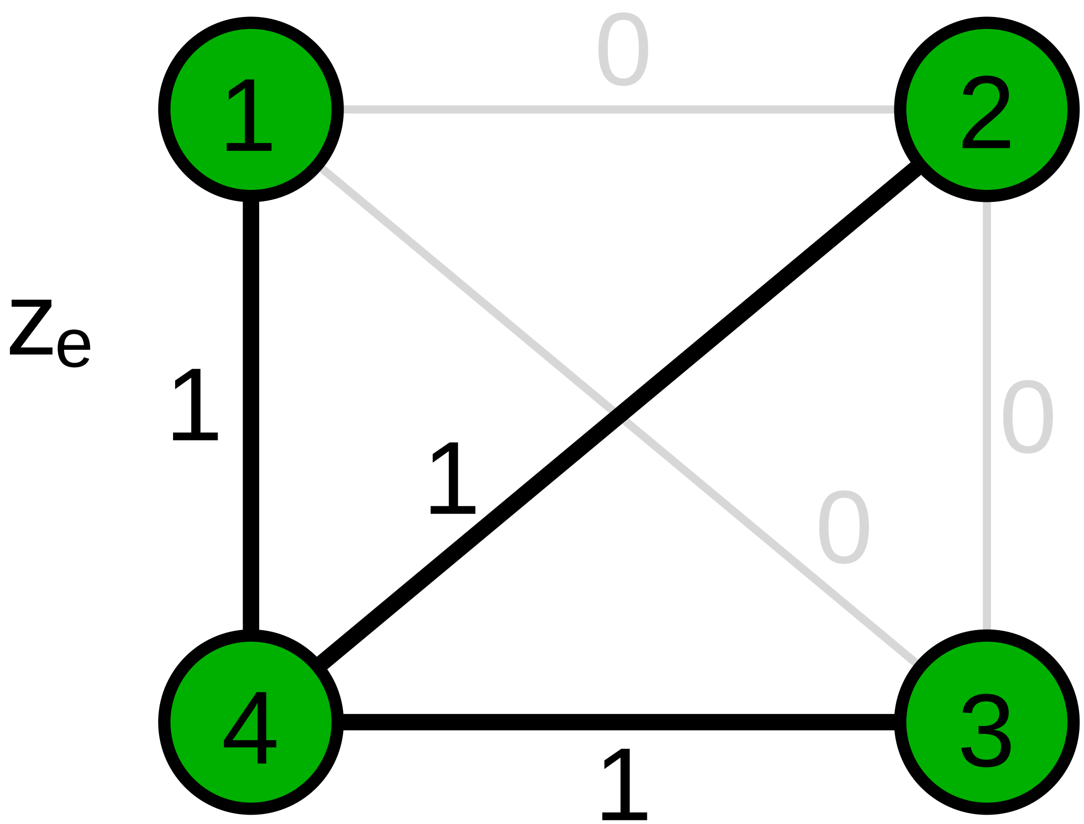
D-F lower bound:

$$\begin{aligned} b(\bar{\mathbf{z}}) &= v(E) + d_{12} + (d_{13} + d_{23}) + 0.d_{13} \\ &= 29 + 4 + 0 + 0 \\ &= 33 \end{aligned}$$

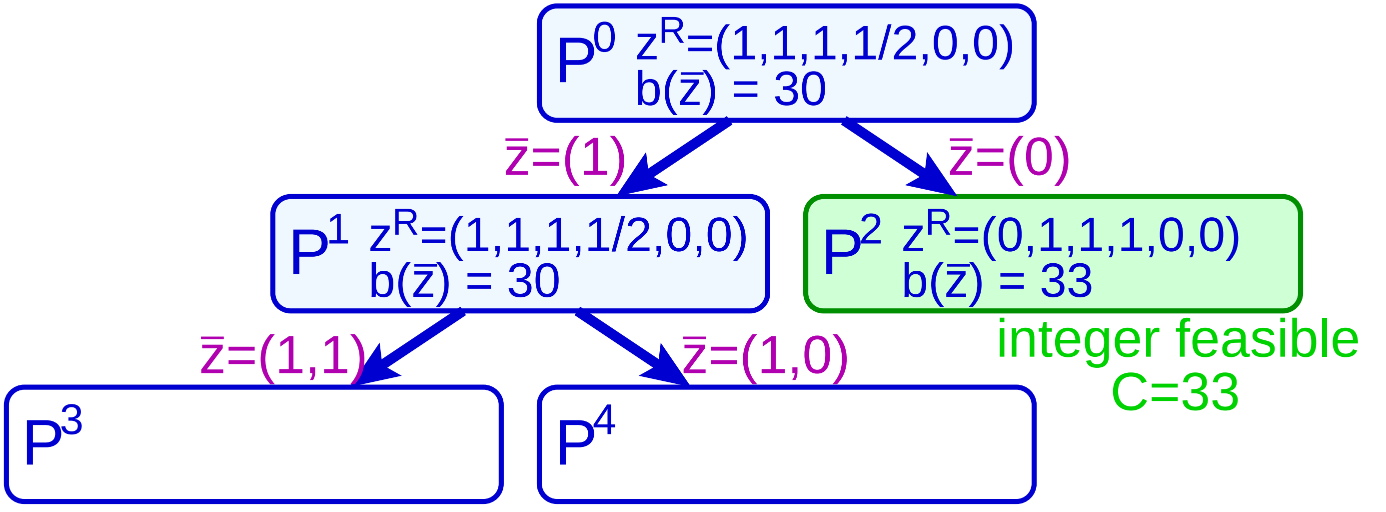
The current value $C = \infty > 33$ so let $C = 33$

Branch and Bound example

P^2 relaxation solution



Branch and Bound example



Branch and Bound example

Problem 3: $\bar{\mathbf{z}} = (1,1)$

Knapsack problem: exactly the same as problem 0 and 1

- solution for P^0 had $z_{12} = z_{34} = 1$, so $\bar{\mathbf{z}} = (1,1)$ doesn't change the solution at all
- $k = 3$, and $z = (1,1,1,1/2,0,0)$
- solution is not integer feasible

D-F lower bound: $b(\bar{\mathbf{z}}) = 30$ (the same as P^0 and P^1)

- the solution was not integer feasible, so we have to branch into two subproblems
  - P^5: $\bar{\mathbf{z}} = (1,1,1)$ (we add the constraint $z_{14} = 1$)
  - P^6: $\bar{\mathbf{z}} = (1,1,0)$ (we add the constraint $z_{14} = 0$)

Branch and Bound example

Problem 4: $\bar{\mathbf{z}} = (1,0)$

Knapsack problem:

- β_{12} is definitely included and β_{34} is excluded,
  - so only consider columns 3-6,
  - and remainder of B is $B - \beta_{12} = 11$
- $\sum_{i=3}^{k} \beta_{e_i} \leq 11$ and $\sum_{i=3}^{k+1} \beta_{e_i} > 11$

$$\beta_{14} + \beta_{24} = 11$$

- $k = 4$, and $z = (1,0,1,1,0,0)$

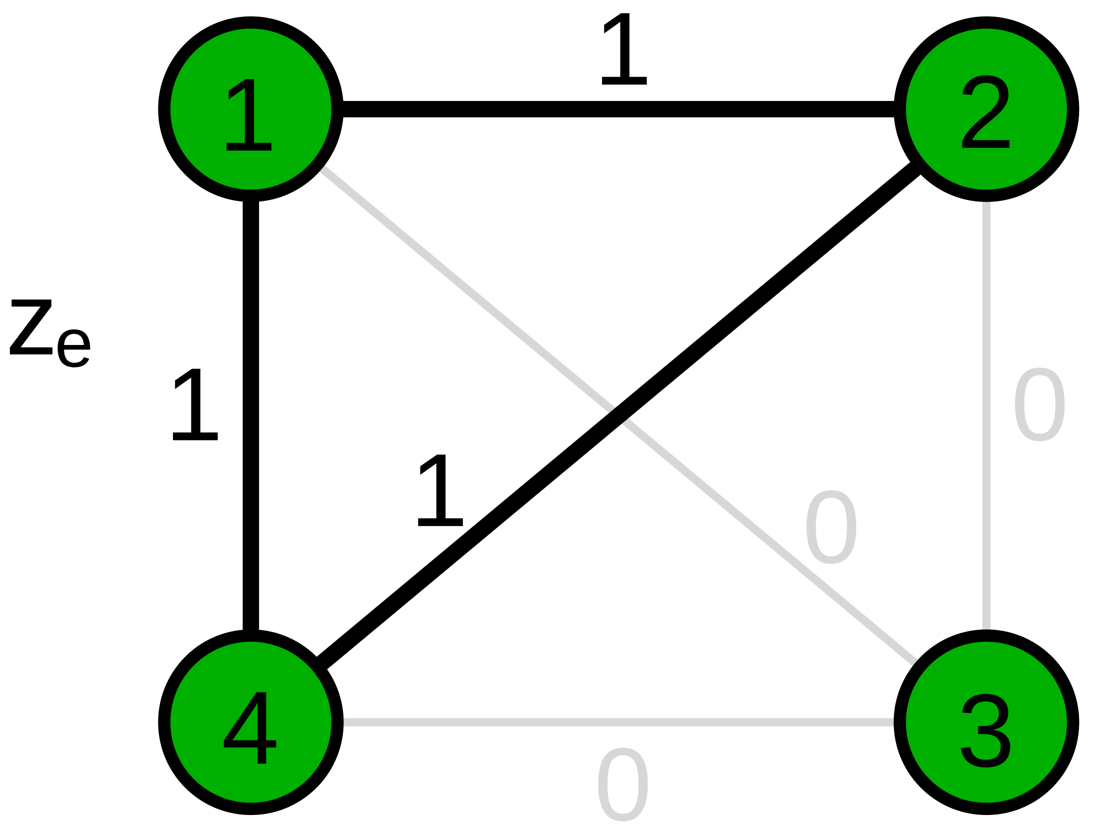
D-F lower bound:

$$\begin{aligned} b(\bar{\mathbf{z}}) &= v(E) + [d_{34} + d_{13} + d_{23}] + 0 \\ &= 29 + 4 = 33 \end{aligned}$$

- solution is integer feasible, so it is **fathomed**
- $b(\bar{\mathbf{z}})$ is too high to be useful though (already $C = 33$)

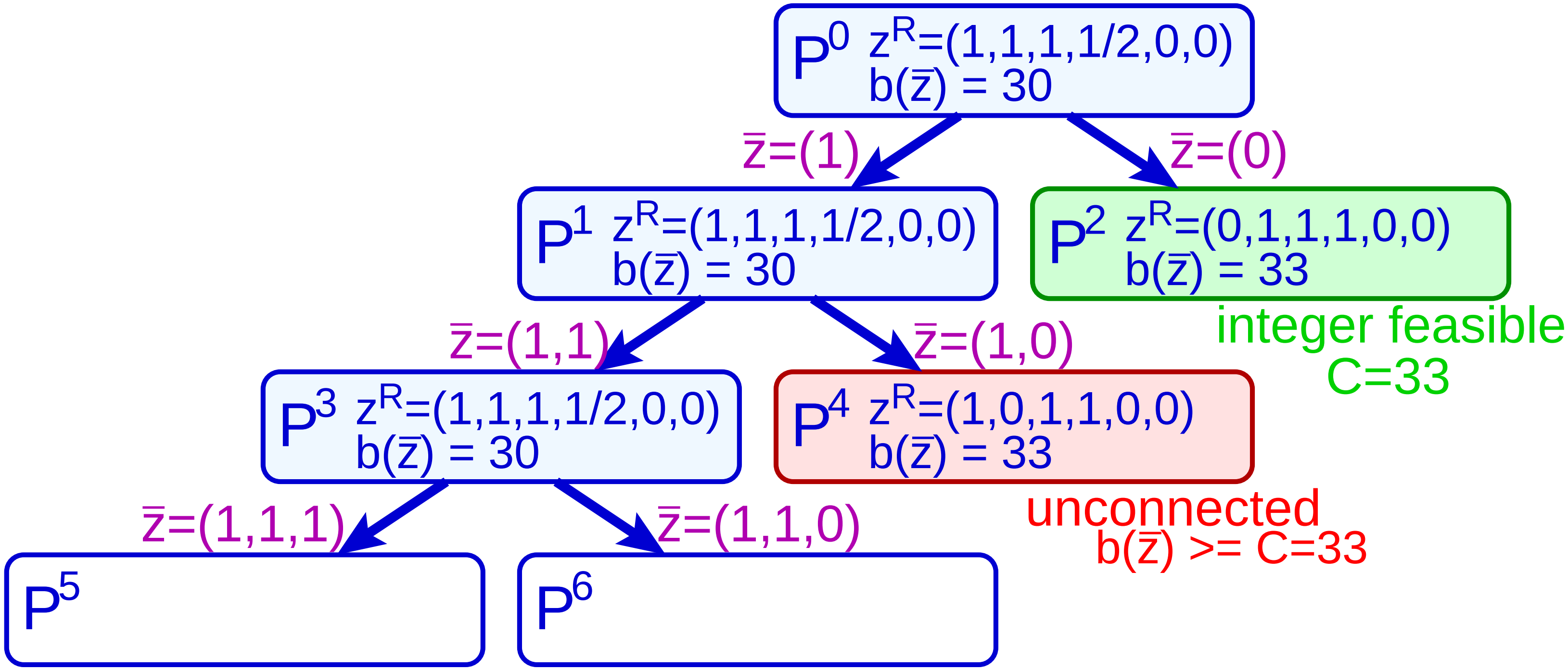
Branch and Bound example

P^4 relaxation solution



Node 3 is NOT connected!

Branch and Bound example



Branch and Bound example

Problem 5: $\bar{\mathbf{z}} = (1,1,1)$

Knapsack problem: exactly the same as problem 0,1 and 3

- solution for P^0 had $z_{12} = z_{34} = z_{14} = 1$, so $\bar{\mathbf{z}} = (1,1,1)$ doesn't change the solution at all
- $k = 3$, and $z = (1,1,1,1/2,0,0)$
- solution is not integer feasible

D-F lower bound: $b(\bar{\mathbf{z}}) = 30$ (the same as P^0, P^1 and P^3)

- the solution was not integer feasible, so we have to branch into two subproblems
  - P^7: $\bar{\mathbf{z}} = (1,1,1,1)$ (we add the constraint $z_{24} = 1$)
  - P^8: $\bar{\mathbf{z}} = (1,1,1,0)$ (we add the constraint $z_{24} = 0$)

Branch and Bound example

Problem 6: $\bar{\mathbf{z}} = (1,1,0)$

Knapsack problem:

- β_{12}, β_{34} are definitely included and β_{14} is excluded,
  - so only consider columns 4-6,
  - and remainder of B is $B - \beta_{12} - \beta_{34} = 8$
- $\sum_{i=4}^{k} \beta_{e_i} \leq 8$ and $\sum_{i=4}^{k+1} \beta_{e_i} > 8$

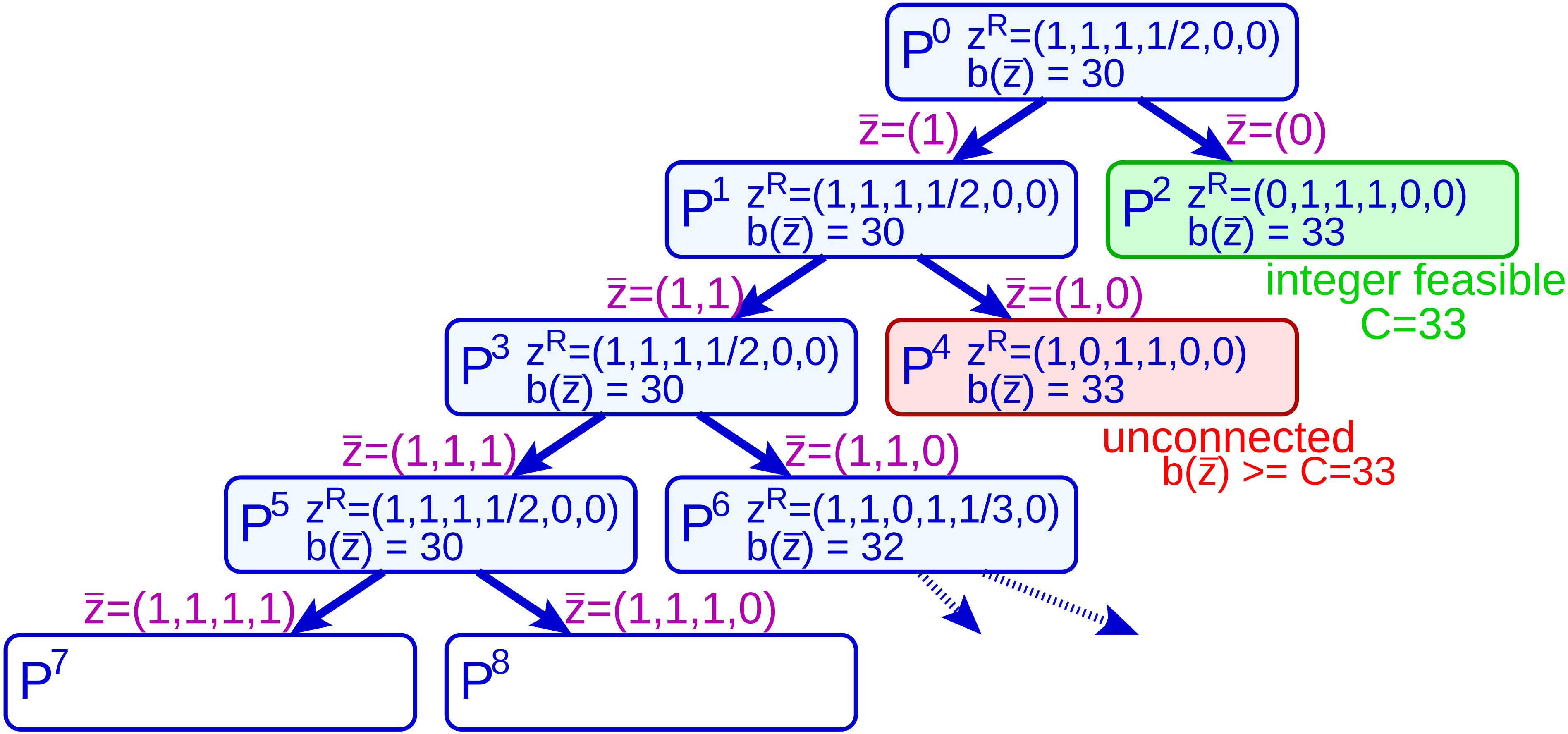
$$\beta_{24} = 6 \qquad \beta_{24} + \beta_{13} = 12$$

- $k = 4$, and $z = (1,1,0,1,1/3,0)$

D-F lower bound: $b(\bar{\mathbf{z}}) = 29 + (3+0) + (1-1/3).0 = 32$

- solution is not integer feasible, so it is not fathomed
- we should branch on this
- lets delay branching for a moment

Branch and Bound example



Branch and Bound example

Problem 7: $\bar{\mathbf{z}} = (1,1,1,1)$

Since $\bar{\mathbf{z}} = (1,1,1,1)$, we include the first four links, so the cost will be at least

$$\sum_{i=1}^{4} \beta_{e_i} = 3+3+5+6 > 14$$

- hence there is no feasible solution
- hence the solution is **fathomed**

Branch and Bound example

Problem 8: $\bar{\mathbf{z}} = (1,1,1,0)$
Knapsack problem:

- $\beta_{12}, \beta_{34}, \beta_{14}$ are definitely included and β_{24} is excluded,
  - so only consider columns 5-6,
  - and remainder of B is $B - \beta_{12} - \beta_{34} - \beta_{14} = 3$
- $\sum_{i=5}^{k} \beta_{e_i} \leq 3$ and $\sum_{i=5}^{k+1} \beta_{e_i} > 3$

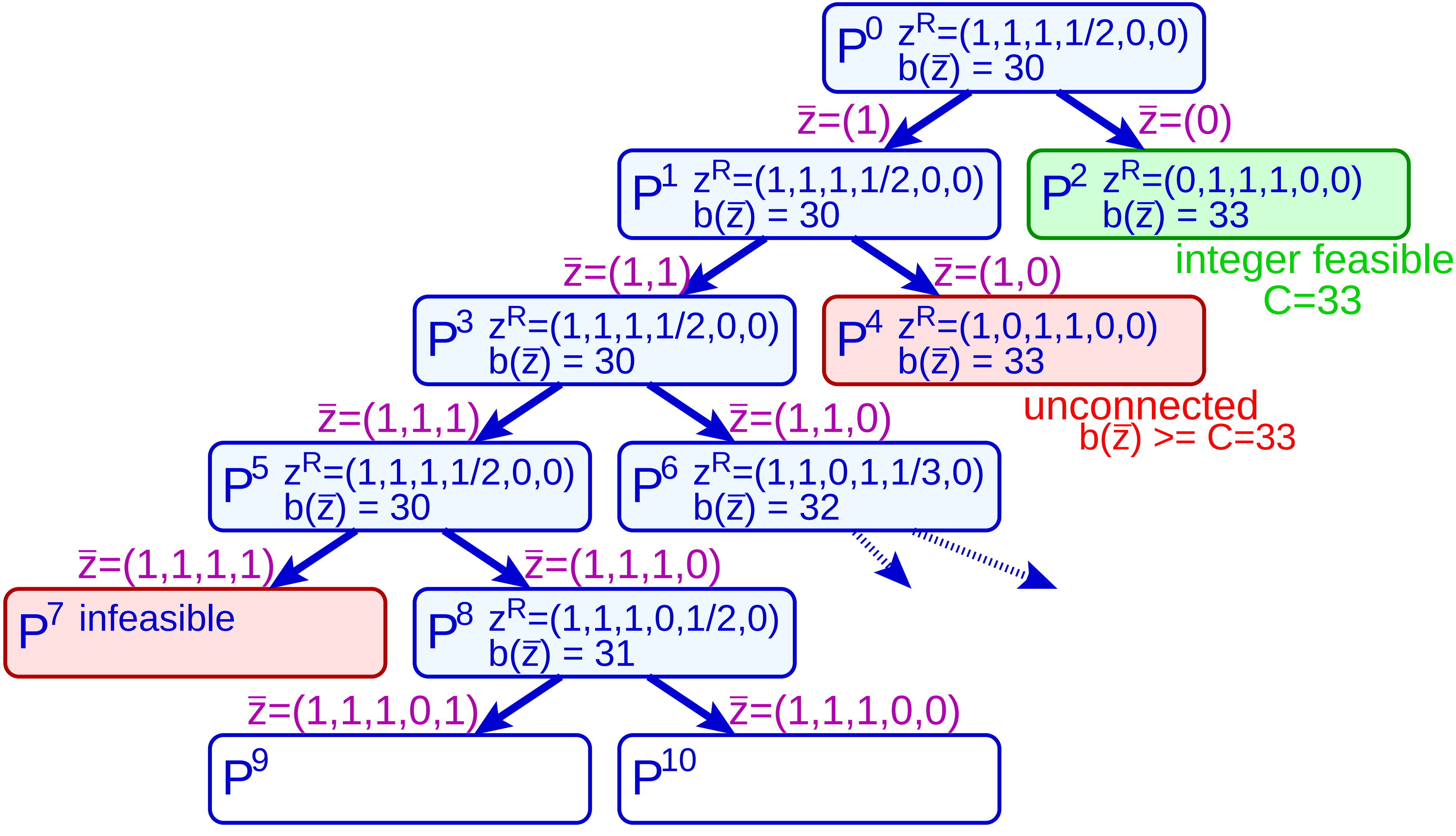
$$\beta_{13} = 6 > 3$$

- $k = 4$, and $z = (1,1,1,0,1/2,0)$

D-F lower bound: $b(\bar{\mathbf{z}}) = 29 + (2+0) + (1-1/2).0 = 31$

- solution is not integer feasible, so it is not fathomed
- we branch on this to get P^9 and P^{10}

Branch and Bound example



Branch and Bound example

Problem 9: $\bar{\mathbf{z}} = (1,1,1,0,1)$

Since $\bar{\mathbf{z}} = (1,1,1,0,1)$, so the cost will be at least

$$\sum_{i=1}^{5} \bar{z}_i \beta_{e_i} = 3+3+5+6 > 14$$

- hence there is no feasible solution
- hence the solution is **fathomed**

Branch and Bound example

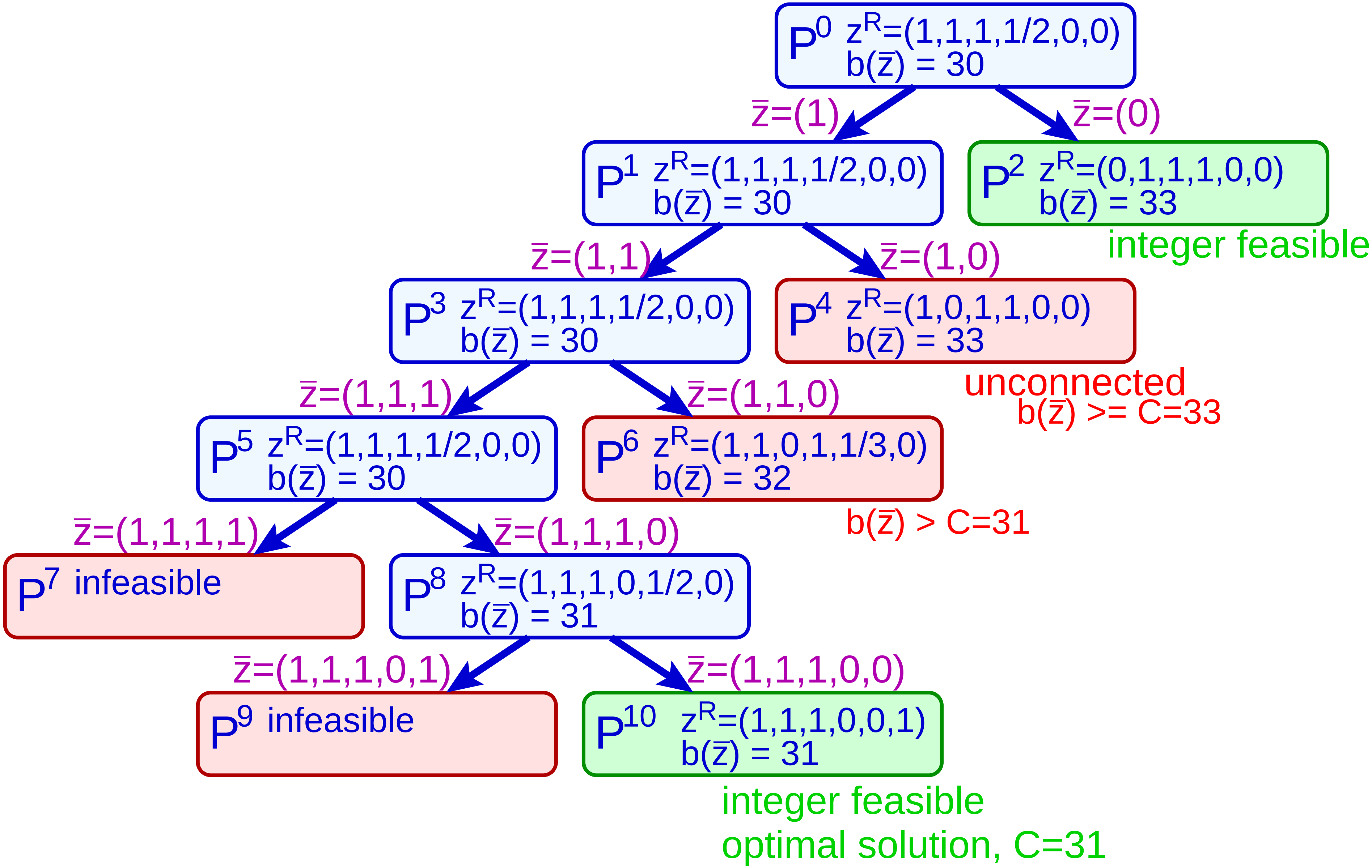
Problem 10: $\bar{\mathbf{z}} = (1, 1, 1, 0, 0)$
Knapsack problem:

- $\beta_{12}, \beta_{34}, \beta_{14}$ are definitely included and β_{24}, β_{13} are excluded,
  - so only consider columns 6,
  - and remainder of B is $B - \beta_{12} - \beta_{34} - \beta_{14} = 3$
- $\beta_{23} = 3$
- $k = 6$, and $z = (1, 1, 1, 0, 0, 1)$

D-F lower bound: $b(\bar{\mathbf{z}}) = 29 + (2+0) + (1-1).d_{24} = 31$

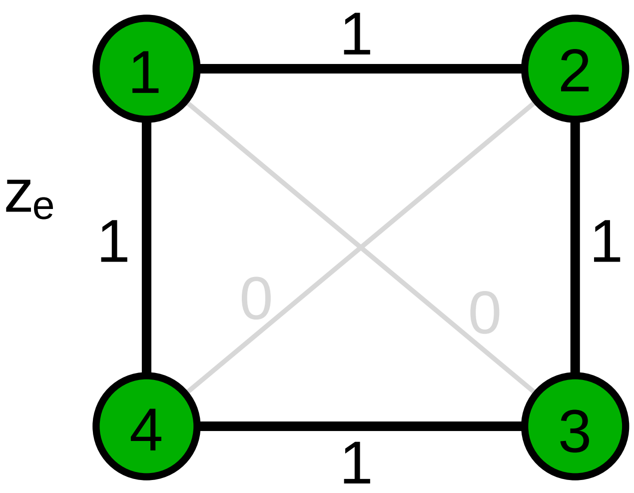
- solution is integer feasible, so it is **fathomed**
- this gives us a new best bound, so $C = 31$, (ditch P^2)
- we can prune P^6 which has $b(\bar{\mathbf{z}}) = 32 > C$

Branch and Bound example



Branch and Bound example

P^{10} solution (and final solution)



- Note this is the same as the result of Minoux's algorithm
- Investment cost for this network expenditure $\sum_e \beta_e = 14$
- We can work out actual operations cost (rather than D-F lower bound)

$$\sum_e c_e(f_e) = \sum_e \alpha_e f_e = 31$$

- Question: what would happen if I made $B = 9$?
- Question: what would happen if I made $B = 1000$?

References

[1] B. Korte and J. Vygen, Combinatorial Optimization. Springer, 2000.

[2] G. Dantzig, "Discrete variable extremum problems," Operations Research, vol. 5, pp. 266–277, 1957.