## **Assignment 6**: Due Monday Oct 28th at 3pm.

Assignments to be handed in through MyUni. Please ensure written assignments are clearly legible. Typed assignments are preferred. Some help may be given in practicals to help get you started with Overleaf/LaTeX in order to present your work well.

This weeks assignment is on Branch and Bound. The following is an example B&B questions, and a solution. Please format your solution the same way to facilitate marking.

**Example Problem:** Solve the following ILP (Integer Linear Program)

$$\begin{array}{rcrcrcrl} \max z & = & 7x_1 & + & x_2 & + & 2x_3 & \\ & & x_1 & + & 2x_2 & + & x_3 & \leq 12 \\ & & 2x_1 & - & x_2 & + & x_3 & \leq 12 \\ & & \frac{5}{2}x_1 & + & x_2 & - & x_3 & \leq 12 \end{array}$$

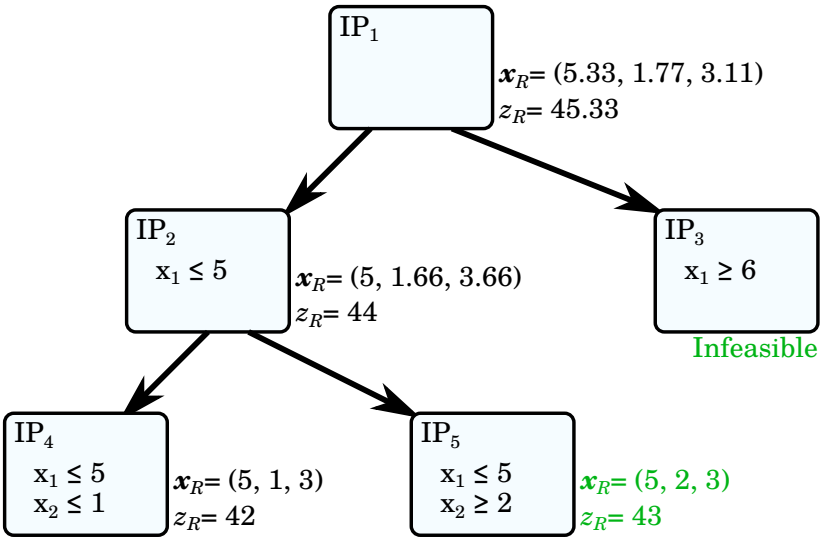
with the $x_i \geq 0$ and integer.

(a) Explain the reason for each *fathoming* at the leaf nodes of the tree. Illustrate your explaination by drawing the tree, and filling in the missing components.

(b) Use the figure (and your working) to derive the optimal solution to the ILP.

**Example Solution:**

(a) The figure below provides complete information about the B&B tree. Note that only additional constraints (above the originals specified in the problem) are shown on the figure. Relaxed problems are solved in the order indicated by their number.




(b) There are three fathomed leaf nodes:

- $IP_3$ is fathomed because its relaxation is infeasible.
- $IP_4$ is fathomed because its relaxation is integer feasible
- $IP_5$ is fathomed because its relaxation is integer feasible.

NB: you don't have to solve the instances using Simplex from scratch. Infeasibility can often be obvious, but also use the trick we learned that branching inequalities can be turned into equalities, reducing the search space.

(c) The best solution is given in $IP_5$, whose relaxation is integer feasible, giving $z = 43$ at $\mathbf{x}^T = (5, 2, 3)$.

1. Solve the following problem using Branch and Bound.

$$\begin{array}{rcrcrcrcl} \max z & = & 10x_1 & + & 4x_2 & + & 9x_3 & & \\ & & 6x_1 & + & 2x_2 & + & 4x_3 & \leq & 17 \\ & & 10x_1 & + & 9x_2 & + & 2x_3 & \leq & 12 \\ & & 1x_1 & + & 8x_2 & + & 10x_3 & \leq & 14 \end{array}$$

   with the $x_i \geq 0$ and integer.

   You may use MATLAB to solve each individual relaxed LP, but you must show all branches of the tree fathomed completely, so you cannot solve the whole problem using MATLAB's `intlinprog`, though you may wish to check the solution that way.

2. In the above problem, what would have happened if we took a "depth-first" search strategy? That is, would we have found the same result, and would it have been as quick?

   [Hint: you do not have to perform the search, only consider what might have happened.]