Tutorial 5

Make sure you prepare these BEFORE the class.

Solutions will be handed out at the tutorial. They will not be put on MyUni.

1. **Translation:** *Ambulance depot location.* The ambulance service needs to locate a number of depots around the city so that it can get to any potential accident within 15 minutes.

   There are M possible locations where we could place a depot. Each location i can reach a subset S_i of the population within the required time. *In the ambulance depot problem, the subsets might be described by regions represented say by circles around the hub, but in general we simply use sets, which makes the problem formulation more general.*

   Each location also has a cost $c_i > 0$. We need choose which potential depot sites to use to minimise the cost of the overall system, bu ensuring that every person is covered by at least one depot.

   Formulate the problem of minimising the cost of the ambulance depots as an Integer Linear Program.

   NB: this is a special instance of the *set-covering problem* (though sometimes in that problem the costs are all equal), so I might call it that in the exam! It applies to many other optimisation problems: most obviously other similar problems such as placement of other emergency services, but also to such diverse areas as optimised supply chains (the sets represent potential suppliers, and the population are the components that need to be supplied); working out how to obtain required expertise for a project; construction of optimal logical circuits; and air-crew scheduling for airlines.

   It is interesting because it was one of the 21 early problems shown to be NP complete in "Reducibility Among Combinatorial Problems", by Richard Karp from Complexity of Computer Computations, pp. 85-103, 1972.

2. **A bit of creativity:** Consider a LP of the following form:

$$\begin{aligned} \max \quad & z = \mathbf{c}^T \mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

   with the addition of lower-bounds on $\mathbf{x}$ of the form:

$$x_i \geq \ell_i \geq 0,$$

   for all i.

   We could solve this by incorporating the lower-bounds into the other constraints, introducing appropriate slack variables, and solving. But this approach is inefficient because we need to introduce many more slack variables and constraints into the problem.

   Instead, change the formulation by making a change of variables to a $\mathbf{x}'$ via a simple linear transformation of the original variables.

3. **Calculations:** Perform Branch and Bound on the following problem:

$$\begin{array}{lrcl} \text{maximise} & 2x_1 - x_2 & & \\ \text{subject to} & x_1 + x_2 & \leq & 7 \\ & 2x_1 - 3x_2 & \leq & 12 \\ & x_1 \geq 0, x_2 \geq 0 & & \\ & x_1, x_2 \text{ integer} & & \end{array}$$

4. **Derivation of the week:** If we use the Simplex algorithm to solve the relaxation steps of Branch and Bound, comment on the overall computational complexity of Branch and Bound.