# MGtoolkit: A python package for implementing metagraphs

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# Abstract

In this paper we present *MGtoolkit*: an open-source Python package for implementing metagraphs - a first of its kind. Metagraphs are commonly used to specify and analyse business and computer-network policies alike. *MGtoolkit* can help verify such policies and promotes learning and experimentation with metagraphs. The package currently provides purely textual output for visualising metagraphs and their analysis results.

*Keywords:* metagraph implementation, computer-network policy, policy analysis

## <sup>1</sup> 1. Motivation

A metagraph is a generalised graph theoretic structure that has several useful applications. They are commonly used to construct and analyse business policies in decision-support systems and workflow-management systems [1]. Metagraphs are also useful to analyse, optimise and troubleshoot communication-network policies [2].

<sup>7</sup> A metagraph is a directed graph between a collection of sets of 'atomic' <sup>8</sup> elements. Each set is a node in the graph and each directed edge represents <sup>9</sup> the relationship between the sets. A simple example is given in Figure 1(a) <sup>10</sup> where multiple sets of users  $(U_1, U_2, U_3)$  are related to sets of network re-<sup>11</sup> sources  $(R_1, R_2)$  by the directed edges  $e_1, e_2$  and  $e_3$  which describes which <sup>12</sup> user  $u_i$  is allowed to access resource  $r_j$ .

In this paper we describe an off-the-shelf tool for implementing metagraphs – *MGtoolkit* – implemented in Python. At the time of writing, we are aware of one other metagraph API- 'Haskell library for metagraph data structure' [3]. This library is being developed in the Haskell programming language but is not complete as far as we can determine.



(a) Metagraph consisting of five sets and three edges.



(b) Metagraph that shows the advantage of a metapath over simple paths.

Figure 1: Metagraph examples.

Developing a metagraph tool faces several key challenges. For instance, 18 a metagraph does not use simple edge weights in its adjacency matrix. Also 19 metagraphs admit representations other than those used for simple graphs, 20 but as in simple graphs, the representation is important for certain algo-21 rithms. In addition, there are many operations defined on a metagraph that 22 must be supported by such a tool. These operations help analyse useful 23 properties such as connectivity, redundancy and allow metagraph transfor-24 mations, but go beyond standard graph operators. 25

Metagraphs have many uses in general. One in particular is in specifying and analysing communication-network policies. We will demonstrate the use of metagraphs here by taking access-control policies in a computer network as an example. But, metagraphs can be equally used in other policy contexts (*e.g.*, QoS, network-service chaining, traffic measurement *etc.*).

## 31 2. Background

<sup>32</sup> The formal structure of a metagraph can be defined as follows:

**Definition 1** (Metagraph). A metagraph  $S = \langle X, E \rangle$  is a graphical construct specified by a generating set X and a set of edges E defined on X. A generating set is a set of variables  $X = \{x_1, x_2, \dots, x_n\}$  and an edge  $e \in E$  is a pair  $e = \langle V_e, W_e \rangle$  such that  $V_e \subset X$  is the invertex and  $W_e \subset X$  is the outvertex.

This definition is similar to that of a directed hypergraph, but in addition metagraphs have several useful operators and properties. One in particular is the notion of a *metapath* [1] which describes connectivity between sets of elements in a metagraph, but is somewhat different from a path in a graph.

<sup>41</sup> **Definition 2** (Metapath). A metapath from source  $B \subset X$  to target  $C \subset X$ <sup>42</sup> in a metagraph  $S = \langle X, E \rangle$  is set of edges E' such that every  $e' \in E'$  is on a <sup>43</sup> path from an element in B to an element in C. In addition  $[\bigcup_{e'} V_{e'} \setminus \bigcup_{e'} W_{e'}] \subseteq$ <sup>44</sup> B and  $C \subseteq \bigcup_{e'} W_{e'}$ .

A metapath is more useful than a simple path (*i.e.*, a sequence of edges). Figure 1(b) illustrates this using two simple paths from  $x_1$  to  $x_5$ :  $(e_1, e_3)$ and  $(e_2, e_3)$ . Element  $x_1$  can reach  $x_5$  without knowing anything about the intermediate nodes  $x_2, x_3, x_4$  if all three edges  $e_1, e_2, e_3$  are used but the simple paths do no capture this fact. But,  $\{e_1, e_2, e_3\}$  does not represent a simple path; there is no sequence of connected edges consisting of these edges. Rather, this metapath is the union of edges in two simple paths.

Reachability between a source node and a target node can be determined by finding valid metapaths between the two in a metagraph [1] (*e.g.*, the metapath from  $x_1$  to  $x_5$  in Figure 1(b) is  $\{e_1, e_2, e_3\}$ ).

<sup>55</sup> Metagraphs have a property called *dominance* which allows to determine <sup>56</sup> whether a metapath has any redundant components (edges or elements) [1]. <sup>57</sup> A metapath is *input-dominant* if no proper subset of its source connects to the <sup>58</sup> target; *edge-dominant* if no proper subset of its edges is also a metapath from <sup>59</sup> the source to the target; and *dominant* if it is both input- and edge-dominant <sup>60</sup> [1]. Non-dominant metapaths indicate redundancies in a metagraph and <sup>61</sup> hence, redundancies in the policies depicted by the metagraph.

In metagraph theory, the notion of cutsets and bridges allow one to locate edges that are critical [1]. A *cutset* is a set of edges which if removed, eliminates all metapaths between a given source and a target. A singleton cutset is a *bridge*. In an access-control policy context for instance, bridges and cutsets indicate if there exists a critical policy or a policy set that enable access between certain users and resources.

It is also possible to derive a projection for a given metagraph. A projection is a simplified metagraph that provides a high-level view of the original metagraph by concealing certain details [1]. In a complex metagraph with many edges, a projection helps to visualise the important aspects with clarity and ease. For instance, in a complex access-control policy with many rules, projections help administrators visualise connectivity between a subset of users and resources.

<sup>75</sup> Metagraphs can have attributes associated with their edges. One such at-<sup>76</sup> tributed metagraph is a *conditional metagraph* [1]. A conditional metagraph <sup>77</sup> includes propositions – statements that may be true or false – assigned to <sup>78</sup> their edges as qualitative attributes [1]. The generating set of these meta-<sup>79</sup> graphs are partitioned into a variables set and a propositions set.

<sup>80</sup> Conditional metagraphs are particularly useful in specifying access-control <sup>81</sup> policies because they allow a policy (such as permit user  $u_1$  to access resource <sup>82</sup>  $r_1$ ) to be activated conditionally (*e.g.*, during business hours only).



Figure 2: *MGtoolkit* entity relationship model (\*-1 denotes a many-to-one relationship and  $\rightarrow$  denotes an extension).

#### <sup>83</sup> 3. Overview of *MGtoolkit*

MGtoolkit is implemented solely in Python 2.7 which is an interpreted, object-oriented, open-source language. Python has a concise but natural syntax for many of its data types, which makes programs exceedingly clear and easy to read; as the saying goes, 'Python is executable pseudocode.' Dependencies of MGtoolkit include the packages NumPy 1.9 and NetworkX 1.7; both very popular and stable open source Python packages.

Figure 2 depicts the entity model we have employed in the underlying framework. Some attributes have been omitted in the *Metagraph* entity for simplicity.

A Metagraph entity consists of a set of Node entities and a set of Edge entities. Each Node contains a subset of elements from the metagraph's generating set. An Edge has the members: invertex and outvertex, assigned a Node each, and an attributes member that returns any edge attributes.

A Metagraph entity also has the methods: add\_edges\_from() and remove\_edges\_from(), to add and delete edges as necessary. In addition, the entity includes methods to derive its adjacency matrix, find metapaths, check metapath properties (*e.g.*, is\_dominant\_ metapath()) and edge properties (*e.g.*, is\_cutset()).

The source and target members of a Metapath return subsets of elements in a metagraph's generating set. The edge\_list member returns an edge set between the source and target which satisfy Definition 2.

A ConditionalMetagraph entity extends a Metagraph and supports proposition attributes in addition to variables. A ConditionalMetagraph inherits

```
1
   # define policy metagraph
\mathbf{2}
   variable_set = {'u1', 'u2', 'u3', 'u4', 'u5', 'u6', 'r1', 'r2', 'r3'}
   propositions_set = {'action=permit', 'action=deny'}
3
4
   cm = ConditionalMetagraph(variable_set, propositions_set)
5
   cm.add_edges_from([
    Edge({'u1','u2','u3'}, {'r1','r2'}, attributes=['action=permit']),
6
    Edge({'u3','u4','u5'}, {'r2','r3'}, attributes=['action=deny']),
7
    Edge({'u2','u3','u5','u6'}, {'r1','r2'}, attributes=['action=permit'])])
8
9
10
   # compute redundancies and conflicts
11
   all_metapaths = cm.get_all_metapaths()
12
   for metapath in all_metapaths:
13
     if cm.has_redundancies(metapath):
         print('redundancy detected: %s'%repr(metapath))
14
15
     if cm.has_conflicts(metapath):
16
        print('conflict detected: %s'%repr(metapath))
```

Listing 1: *MGtoolkit* implementation of policy in Figure 1(a).

Listing 2: Partial output from running code in Listing 1.

the base properties and methods of a Metagraph and additionally supports methods to derive its context metagraphs (*i.e.*, get\_context()), check connectivity properties (*e.g.*, is\_fully\_connected()) and redundancy properties (*e.g.*, is\_non\_redundant()).

The code snippet in Listing 1 instantiates the example access-control policy in Figure 1(a) using *MGtoolkit* and then checks policy consistency. It returns a redundancy and two conflicts—one is shown in Listing 2. The redundancy is due to  $e_1$  and  $e_3$  both enabling access to  $R_1$  from  $u_2$  and  $u_3$ . The conflicts stem from  $e_3$  denying access to  $R_2$ . More detailed examples based on business policies and workflows can be found on pages 81, 109 and 126 of the metagraph text [1].

## <sup>118</sup> 4. Impact and challenges

There are many packages available for analysing graphs, *e.g.*, igraph, NetworkX, Gephi [4, 5, 6]. These are being increasingly utilised. Metagraphs provide a powerful generalisation of simple graphs and are particularly suitable for modeling business and computer-network policies [1, 2].

*MGtoolkit* is the first publicly available Python API for implementing 123 metagraphs. It serves two key purposes. Firstly, the API allows users to 124 learn about metagraphs in an interactive manner by creating metagraph 125 examples, applying metagraph operations and evaluating the results. The 126 documentation and tutorials associated with the package simplify the learn-127 ing curve. Secondly, the API is a building block for developing and analysing 128 metagraph-based applications such as decision-support systems. Developers 129 can harness the advantages and power of metagraphs in to their applications 130 by simply importing *MGtoolkit*. 131

We believe our API is a first step to revisit old questions and tackle new challenges. For instance, in the specification and analysis of computer network policies: current approaches either lack high-level specification capability or formal semantics. *MGtoolkit* is a gateway to harness the best of both of these worlds.

<sup>137</sup> We have used the GitHub open source code hosting and development <sup>138</sup> platform to enable user collaboration.

A key drawback in developing *MGtoolkit* was the fact that the only meta-139 graph text available for reference contained several discrepancies. For in-140 stance, the inverse metagraph generation algorithm given in the text failed 141 to replicate the example output provided (Figure 4.9 on page 47 in [1]). Upon 142 clarification with the author, we found that the example was in fact incorrect. 143 Also several metapath examples given contradicted the definition of a 144 metapath (e.g., metapath  $M_4$  on page 28 in [1]). We strictly adhered to the 145 definition because the formal metagraph properties derived were based on 146 the definition. 147

### <sup>148</sup> 5. Conclusions and future work

In this paper, we present *MGtoolkit*: an open-source Python package for implementing metagraphs. The software promotes learning and experimentation with metagraphs and can help analyse business- and computer-network policies alike.

In the future, we are planning several applications based on *MGtoolkit*, one in particular is a tool for the formal analysis of computer-network policies. Additionally, some of the algorithms suggested in [1] are not efficient and we plan to improve on them.

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# 174 Current code version

Nr.	Code metadata description	
C1	Current code version	V1.0.1
C2	Permanent link to code/repository	https://github.com/
	used for this code version	dinesharanathunga/mgtoolkit
C3	Legal Code License	MIT
C4	Code versioning system used	git
C5	Software code languages, tools, and	Python2.7
	services used	
C6	Compilation requirements, operat-	Mac OS X, Linux
	ing environments & dependencies	
C7	If available Link to developer docu-	https://readthedocs.org/
	mentation/manual	projects/mgtoolkit/badge/
		?version=latest
C8	Support email for questions	mgtkhelp@gmail.com

Table 1: Code metadata